# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: UGMM-105 | Course Title**: Mechanics-I (Statics and Dynamics)** | Maximum Marks : 30 |
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**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If $T$ be the tension at any point P of a common catenary and $To$ be the tension at the lowest point $A$ then prove that $T^2 - To^2 = W^2$ $when\ W\ in\ the$ weight of the are AP of the cetenery.
2. Five weight less rods of equal length are joined together so as to from a rhombus ABCD with one diagonal *BD*. at a weight $W$ be attached to $C$ and the system be suspended from $A$ then show that there is a thrust in *BD* equal $w/\sqrt{3}$ .
3. The velocities of a pastiche along and perpendicular to the radius vector from a fixed point are $\lambda\ r$ & $\mu\theta$. Find the path of the particle.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. A particle is allowed to move from the top of a cycloid whose vertex is upward and plane vertical with negligible velocity. Find the point where the particle leaves the cycloid.
5. A body consisting at a core and a hemisphere on the same base rests on a rough horizontal table the hemisphere being in contact with the table of the height of the cone is $\sqrt{3}$ times the radius of the hemisphere . Find whether the equilibrium will be stable or unstable.
6. A particle moves with a central acceleration which varies inversely as the cube of the distance if it is projected from an apse at a distance a from the origin with velocity which is $\sqrt{2}$ time of the velocity for a circle of radius a then show that its path is $r\ \cos\frac{\theta}{\sqrt{2}} = a.$
7. A particle whose mass is m is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin if it stats from rest a distance a then show that it will arrive at the origin is time $\frac{\pi}{4\sqrt{\mu}}$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: UGMM-106 | Course Title**: Mechanics-II (Dynamics and Hydrodynamics)** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the moment of inertia of a rod of length $2a$& mass M about a line through its centre perpendicular to $its$ length.
2. Find the moment of inertia of a circular disc of radian $'a'$ about $its$ diametre.
3. At the vertex c of a tangle ABC which is a right angle at c show that the principle axis in the plane are inclined to the sides at an angle $\frac{1}{2}\ tan^{-1} \frac{ab}{a^2-b^2}$.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :**Answer each question in 200 to 300 Words. All carry equal marks.

4. One end of a light string is fixed to a point of the rim of a uniform circular disc of radian 'a' & mass 'm' and the string is wound several times round the rim. the free end is attached to a fixed point and the disc is held so that the part of the string not in contact with the vertical of the disc be let go find the acceleration & tension of the string.
5. Find the moment of inertia of a right circular cylinder about a straight line through its centre of gravity perpendicular to its axis.
6. A straight uniform rod can turn freely about one end O, hangs from O vertically. Find the least angular velocity with which it must begin to moves so that it may perform complete revolution in a vertical plane.
7. Show that the moment of inertia of the area bounded by $r^2 = a^2 cos\ 2\theta$ about its axis is $\frac{Ma^2}{16}(\pi - 8/3\ )$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: UGMM-107 | Course Title**: Linear Algebra** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find all eign values and eign vectors of a linear transformation
   $T : IR^3 IR^3, \ defined\ as\ \ T\,(x, y, z) = (2x + y,\ y - z, 2y + 4z)$. Is T diagonolizatble
2. If $w_1$and $w_2$are any two finite subspaces of a vector space V then show that
   $$dim\,(w_1 + w_2) = dim\,w_1 + dim\,w_2 - dim\,(w_1 \cap w_2)$$
3. Find the eigen Values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Let V be a vector space over a field F such that it has no proper subspace. Then show that either
   $$V = \{\,o\,\}\ or\ dim\,V = 1.$$
5. Which of the following is a linear transformation where $T : IR^2 \rightarrow IR^2$
   (a) $T\,(x_1, x_2) = (1 + x_1, x_2)$
   (b) $T\,(x_1, x_2) = (x_2, x_1)$
6. A function f is defined on $IR^2$ as follows:
   $$f\,(x, y) = (x_1 - y_1)2 + x_1 y_2\,, where\ x = (x_1 - x_2)\ and\ y = (y_1, y_2)$$
   $$Is\ f\ a\ bilinear\ forms?\ Verify.$$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: UGMM-108 | Course Title**: Calculus of function of several variable and Vector Calculus** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. at $u = e^{xyz}$ $then\ show\ that\ \frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)$ is it also equal to $\frac{\partial^3 u}{\partial y \partial z \partial x}$ ?
2. Show that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$
3. A particle moves so that its position vector in given by $\bar{r} = \hat{\imath}\cos wt + \hat{\jmath}\sin wt$ Show that the velocity $\bar{v}$ is perpendicular $\bar{r}$ and $\bar{r} \times \bar{v}$is constant vector.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Find the deviational derivative of $f(x) = \mathrm{xy}^2 + \mathrm{yz}^{\,3}$ at the point $(1, -1, 1)$ along the vector $\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$
5. at $\mathrm{u} = \tan^{-1}\left(\frac{\mathrm{x}^3+\mathrm{y}^3}{\mathrm{x}-\mathrm{y}}\right)$ then show that $\mathrm{x}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \mathrm{y}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = \sin 2\mathrm{u}$
6. Determine the point where the function $\mathrm{x}^4 + \mathrm{y}^4 - 2\mathrm{x}^2 + 4\mathrm{xy} - 2\mathrm{y}^2$ has a maximum are minimum.
7. Find curl (curl $\bar{\mathrm{F}}$) at the point (0,1,2) where $\bar{\mathrm{F}} = (\mathrm{x}^2\mathrm{y})\hat{\imath} + (\mathrm{xyz})\hat{\jmath} + (\mathrm{z}^2\mathrm{y})\hat{\mathrm{k}}$

   Or

   Evaluate $\int \bar{F}.d\bar{r}\ \ whre\ \ \bar{F} = (3x^2)\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the straight line joinery (0,0,0) & (2,1,3)

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: DCEMM-109 | Course Title**: Abstract Algebra** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

**NOTE: Answer each question in 500 to 800 words. All carry equal marks.**

Maximum Marks: 18

1. State and Prove fundamental theorem of group homomorphism.
2. Let N be a normal subgroups of a group G and H be a subgroup of G then show that:

   (i) H ∩ N is normal subgroup of H (ii) HN is a subgroup of G (iii) N is normal subgroup ofHN.
3. Prove that if G is abelian then G| Z(G) is cyclic where Z(G) is centre of G.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note: Answer each question in 200 to 300 Words. All carry equal marks.**

4. Give all sub groups of ($Z_{12}$, +)
5. Let $f\colon G_1 1 \rightarrow G_2$be a group homomorphism then show that kernel f is a normal subgroup of $G_1$.
6. Give an example non-cycle group whose all subgroups are cyclic.
7. Find all zero divisor elements of $Z/20$.

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| | | |
|---|---|---|
| Course Code: DCEMM-110 | Course Title**: Number Theory** | Maximum Marks : 30 |

**(Section 'A')**

(Long Answer Questions)

**NOTE: Answer each question in 500 to 800 words. All carry equal marks.**

Maximum Marks: 18

**1.** Find the remainders obtained on division of the following:

(a) $3^{50}$ by 101 (b) $159^{7654}$ by 23

**2.** Find the g.c.d. of 163 and 34 and express it in the form $163m + 34n$ in two ways.

**3.** Prove that (a) $18! + 1 \equiv 0 \pmod{437}$ (b) $28! + 233 \equiv 0 \pmod{899}$.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note : Answer each question in 200 to 300 Words. All carry equal marks.**

**4.** Show that every square is congruent to 0 or 1 (mod 8).

**5.** Find the value of $\emptyset(m) if\ m = 500$.

**6.** Find the following Legendre symbols: (a) $\left(\frac{19}{41}\right)$ (b) $\left(\frac{3}{7}\right)$ (c) $\left(\frac{5}{11}\right)$ (d) $\left(\frac{6}{11}\right)$

**7.** Find the value of Mobius function $\mu(n)$ for $n$

(a) 15 (b) 30 (c) 47 (d) 100

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: DCEMM-112 | Course Title**: Advance Analysis** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

**NOTE: Answer each question in 500 to 800 words. All carry equal marks.**

Maximum Marks: 18

1. Every Cauchy sequence $(S_n)$ of real Numbers converges.
2. Let $(X_l, d_l)$ and $(X_2, d_2)$ be two discrete metric spaces. Then verify that the product metric on $X_1 \times X_2$ is discrete.
3. Show that a Cauchy sequence is convergent⇔ it has a convergent subsequence.
4. Let $(X, d)$ be a metric space and $A \subseteq X$.Show that $\bar{A} = \{x \in X: d(x, A) = 0\}$.

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note : Answer each question in 200 to 300 Words. All carry equal marks.**

**5.**Define Complete Metric Space. Given an example of a metric space which is not Complete.

**6.** Any compact metric space is totally bounded.

**7.** Statement and Prove Mean value theorem.

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: DCEMM-113 | Course Title**: Function of Complex Variable** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If $u = \frac{1}{2}\log(x^2 + y^2)$, find $v$ such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of $z$.
2. Find the radius of convergence R of the following power series:

   $(i)\ \sum_{n=0}^{\infty} z^n$ (ii) $\sum_{n=1}^{\infty} \frac{z^n}{n}$ (iii) $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$
3. Using Cauchy integral formula, calculate the following integrals.

   $\int_c \frac{\cos(\pi z)}{z(z^2+1)} dz$, where $C$ is the circle $|z| = 2$
4. Evaluate $\int_0^{3+i} z^2 dz$ along the line joining the points (0, 0) and (3, 1).

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

5. Evaluate $\int_{\boldsymbol{c}} \frac{\boldsymbol{dz}}{\boldsymbol{z-2}} for\ n = 2,3,4$ ...where $z = a$ is a point inside the simple closed curve c.

**6.** Find Taylor Series of $f(z) = \frac{1}{z}$ about $z = -1, z = 1$ $and$ $z = 2$. Determine the circle of convergence in each case.

**7.** For the conformal transformation $w = z^{2}$. Show that the circle $|z - 1| = 1$ transforms into the cardioid $R = 2(1 + cos\emptyset)$ where $w = Re^{i\theta}$ in the $w$ −plane.

# Uttar Pradesh Rajarshi Tandon Open University

School of Science,Assignment Session 2023-24

| Course Code: SBSMM-03 | Course Title**: Elementary Analysis** | Maximum Marks : 30 |
|---|---|---|

**(Section 'A')**

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

**1.** Write truth tables fo the sentence $\mathrm{P} \Rightarrow \mathrm{P}$ and

$\mathrm{P} \Rightarrow -\mathrm{P}$. Is the First sentence a tautology.

**2.** The diagonal or the equality relation & $in\ a\ set\ S\ is\ an\ equivalence$

relation in S. For it $\mathrm{x, y} \in \mathrm{S}$ the x y iff $\mathrm{x = y}$.

**3.** Let x be a set. Consider the relation R in $\left(\mathrm{e(x)}\right)$, given by : for A, B $\in \left(\mathrm{e(n)}\right)$ARB if $\mathrm{A} \subseteq \mathrm{B}$.

**4.** $Let\ f: X \to Y\ be\ a\ map\ and\ let\ A\ and\ B\ subsets\ of\ X, then\ A \subseteq B \Rightarrow f(A) \subseteq f(B)$

**(Section – B)**

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

$$\mathbf{5}.\ Let\ X = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y = [-1,1]$$

$Let\ f: X \to Y\ given\ by\ f(x) = sinx, x \in X.$

**6.** Evaluate $\iint xy\, dxdy$ over the region in the positive quadrant for which $x + y \leq 1$.

**7.** Find the volume inside the paraboloid $x^2 + 4z^2 + 8y = 16$ and on the positive side of $xz$ –plane.